

[Time:2.30 Hrs]

[ Marks:75 ]

Please check whether you have got the right question paper.

- N.B: 1. All question are compulsory.  
2. Figures to the right indicate full marks.

**Q.1 Attempt any four of the following: 20**

- A Define complex number, Conjugate, Modulus and Argument of a complex number. Give one example for all.
- B Define GF (2). Explain with an example how addition and multiplication is carried out in GF (2).
- C Solve:  $5x_1 = 15$ ,  $4x_1 + 2x_2 = 10$ ,  $3x_1 + 5x_2 + 2x_3 = 18$
- D Define Linear combination. Express  $W = (6, -2, 5)$  as a linear combination of  $v_1 = (-2, 1, 3)$  and  $v_2 = (3, 1, -1)$  and  $v_3 = (-1, -2, 1)$ .
- E Define Vector Space.
- F For the given pairs of vectors, find vector  $u + v$ ,  $u - v$ ,  $v - u$ ,  $2u + 3v$ ,  $-2u - 7v$  where  $u = (2, 5)$  and  $v = (4, -1)$

**Q.2 Attempt any four of the following: 20**

- A Define Row space and Column space of matrix. Hence find the same for:

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 4 & 5 \\ -2 & 3 & 1 \end{bmatrix}$$

- B Define: 1) Matrix-Vector Multiplication 2) Vector- Matrix Multiplication
- C Let F be any field. Define function from  $F^2$  to F by  $f(x, y) = x - y$ . Show that it is a linear function.
- D Explain two greedy algorithms for set of generators
- E Check if the vectors  $[1, 0, 0]$ ,  $[0, 3, 0]$ , and  $[3, 9, 0]$  are linearly dependent
- F State and prove Kernel-Image theorem.

**Q.3 Attempt any four of the following: 20**

- A Define Echelon Form of a matrix. Give example.
- B Solve the following system of equations by Gaussian elimination method:  
 $2x + y + z = 10$ ;  $3x + 2y + 3z = 18$ ;  $x + 4y + 9z = 16$
- C Find the basis for null spaces of the following matrix:

$$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 4 & -1 & 1 & -1 \\ 8 & -2 & 3 & -1 \end{bmatrix}$$

- D Define inner product and state its properties.  
 E If  $u = (1, -3, 5)$  and  $v = (3, 1, -4)$ , find the inner product of  $u$  and  $v$ . Also find norm of  $u$ , norm of  $v$ , and norm of distance between  $u$  and  $v$   
 F State and prove Cauchy-Schwartz inequality.

**Q.4 Attempt any three of the following:**

**15**

- A Find the square root of  $Z = 3-4i$   
 B Let  $u = 11001$  and  $v = 10110$  are two vectors over  $GF(2)$ , find their dot product.  
 C Suppose  $M = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$  and  $w = [2, -1]$

Compute matrix – vector multiplication in terms of dot product.

- D Define basis. Check if  $\{(1,0), (1,1)\}$  is a basis for  $\mathbb{R}^2$   
 E Find the orthogonal Complement of  $W = \text{span}\{w_1, w_2\}$ , where  $w_1 = (3, 0, 1, 1)$  and  $w_2 = (0, 2, 5, 1)$ .  
 F Write a note on Internet worm.

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